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"Exact" Low Gain Formulation of the Free Electron Laser-Including Transverse Velocity Spread and Wiggler Incoherence.

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It is shown that in the small signal-low gain regime, an analytic expression for the gain can be derived, which is correct for general electron distribution function and general longitudinal variation of the transverse magnetic field pump. This expression is used to evaluate the effects on the gain curve due to transverse momentum spread in the beam as well as inaccuracies (incoherence) in the magnetic pump phase and amplitude. The restricting criteria for the neglect of these effects are derived.

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"EXACT" LOW GAIN FORMULATION OF THE FREE ELECTRON LASER - INCLUDING TRANSVERSE VELOCITY SPREAD AND WIGGLER INCOHERENCE

The linear analysis of the bremsstrahlung free electron laser was presented first by Madey [1], and was further elaborated by a number of authors [2-4]. In this paper we will show that in the low gain regime (which is of interest mostly for the application of oscillators) it is possible to obtain an explicit gain expression for an arbitrary transverse magneto-static pump and arbitrary velocity distribution of the electron beam. As a special case we will use this expression to find the effect of random perturbation in the periodic pump field and transverse velocity spread on the gain of the free electron laser.

The analysis is based on the technique used by Sprangle et al [4] to solve the free electron laser equations. The linearized Vlasov equation is used to find the perturbation to the beam velocity distribution function $f^{(1)}$ $(\underline{p}, z, t)$ caused by the pump field $B_o(z)$ and the optical field $\underline{E}(r,t)$, $\underline{B}(r,t)$:

$$\frac{\partial f^{(1)}}{\partial t} + v_z \frac{\partial f^{(1)}}{\partial z} - \frac{e}{c}\left[\underline{v} \times B_o(z)\right] . \quad \frac{\partial f^{(1)}}{\partial \underline{p}} = \tag{1}$$

$$= e\left[\underline{E}\ (z,t) + \underline{v} \times \underline{B}\ (z,t)/c\right] \frac{\partial f^{(o)}}{\partial \underline{p}}$$

where $f^{(o)}$ $(\underline{p},z)$ is the beam distribution function in the static magnetic field $B_o(z)$ in the absence of the radiation field.

The transverse magnetic pump field $\underline{B}_o(z)$ can be described in terms of a vector potential $\underline{A}_o(z)$:

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$$\underline{B}_o(z) = \underline{\nabla} \times \underline{A}_o(z) \tag{2}$$

$$\underline{A}_o(z) = A_{ox}(z)\ \hat{e}_x + A_{oy}(z)\hat{e}_y \tag{3}$$

and following [4] we express Eq. 1 in terms of the three constants of motion of the electrons in the static field $\underline{A}_o(z)$:

$$\alpha(p_x,\ z) = p_x - \frac{e}{c}\ A_{ox}(z) \tag{4}$$

$$\beta(p_y,\ z) = p_y - \frac{e}{c}\ A_{oy}(z) \tag{5}$$

$$u(p_x,p_y,p_z) = (p_x^2 + p_y^2 + p_z^2)^{1/2} \tag{6}$$

where $e = |e|$. α and β are the x and y components of the canonical momentum, and u is the total momentum of the electron. These variables are independent of z. Eq. (1) then simplifies into

$$\frac{\partial g}{\partial t}^{(1)} + v_z(\alpha,\beta,u,z,)\ \frac{\partial g}{\partial z}^{(1)} = e[\underline{E}(\underline{r},t) + \frac{1}{c}\ \underline{v}(\alpha,\beta,u,z) \times \underline{B}(\underline{r},t)].$$

$$[\frac{\partial g}{\partial \alpha}^{(o)}\ \hat{e}_x + \frac{\partial g}{\partial \beta}^{(o)}\ \hat{e}_y + \frac{\underline{p}(\alpha,\beta,u,z)}{u}\ \frac{\partial g}{\partial u}^{(o)}] \tag{7}$$

where $g^{(o)}\ (\alpha,\beta,u) = f^{(o)}\ (\underline{p},z), g^{(1)}\ (\alpha,\beta,u,z,t) = f^{(1)}\ (\underline{p},z,t)$ and

$$\underline{v}(\alpha,\beta,u,z) = \underline{p}(\alpha,\beta,u,z)/[m_o\gamma_o(u)].$$

The detailed analysis in [4] indicated that the contribution of the first two terms in the second parenthesis in Eq.7 is negligible in comparison to the contribution of the third term. Hence, for the sake of simplicity we will neglect them from now on (this approximation is implicitly used in all the other analyses [1-3].

We assume that the electron beam plasma is tenuous enough so that the electrostatic potential $\Phi(z)$ due to space charge effects can be neglected. We also assume that the electromagnetic field is uniform across the electron beam cross section. The electromagnetic field can be then expressed in terms of circularly polarized vector potentials $\tilde{A}_+(z)$ and $\tilde{A}_-(z)$

$$\underline{E}(z,t) = -\frac{1}{c}\frac{\partial \underline{A}}{\partial t} = \frac{i\omega}{c}[\tilde{A}_+(z)\hat{e}_+ + \tilde{A}_-(z)\hat{e}_-]e^{-i\omega t} + \text{c.c.} \tag{8}$$

$$\underline{B}(z,t) = \underline{\nabla} \times \underline{A} = i\,[-\frac{d}{dz}\tilde{A}_+(z)\hat{e}_+ + \frac{d}{dz}\tilde{A}_-(z)\hat{e}_-]e^{-i\omega t} + \text{c.c.} \tag{9}$$

where $\hat{e}_+ = \frac{1}{2}(\hat{e}_x + i\hat{e}_y)$, $\hat{e}_- = \frac{1}{2}(\hat{e}_x - i\hat{e}_y)$ are respectively the right and left hand circular polarization unit vectors, and $\tilde{A}_+ = \tilde{A}_x - i\tilde{A}_y$, $\tilde{A}_- = \tilde{A}_x + i\tilde{A}_y$.

With all these assumptions, Eq. 7 now simplifies into

$$-i\omega\tilde{g}^{(1)}(\alpha,\beta,u,z) + v_z(\alpha,\beta,u,z)\frac{\partial}{\partial z}\tilde{g}^{(1)}(\alpha,\beta,u,z) =$$

$$= \frac{ie\omega}{2c}[p_-(\alpha,\beta,u,z)\ \tilde{A}_+(z) + p_+(\alpha,\beta,u,z)\tilde{A}_-(z)]\ \frac{1}{u}\frac{\partial}{\partial u}\ g^{(o)}(\alpha,\beta,u) \tag{10}$$

where $g^{(1)}(\alpha,\beta,u,z,t) \equiv \tilde{g}^{(1)}(\alpha,\beta,u,z)e^{-i\omega t} + \text{c.c.}$ and $p_{\pm} = p_x \pm ip_y$.

The perturbation in the distribution function gives rise to a transverse current

$$\underline{J}(z,t) = [\tilde{J}_x(z)\hat{e}_x + \tilde{J}_y(z)\hat{e}_y]e^{-i\omega t} + c.c. = [\tilde{J}_+(z)\hat{e}_+ + \tilde{J}_-(z)\hat{e}_-]e^{-i\omega t} + c.c. \tag{11}$$

where

$$\tilde{J}_{\pm}(z) = -e\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty} \frac{u\,d\alpha\, d\beta\, du}{p_z(\alpha,\beta,u,z)} v_{\pm}(\alpha,\beta,u,z)\tilde{g}^{(1)}(\alpha,\beta,u,z) \tag{12}$$

$$v_{\pm} = \frac{p_{\pm}}{\gamma(u)m} \tag{13}$$

$$p_{\pm} = p_x \mp ip_y = \alpha \mp i\beta - \frac{e}{c}(A_{ox} \mp iA_{oy}) \tag{14}$$

$$p_z = [\,u^2 - (\alpha + \frac{e}{c}A_{ox})^2 - (\beta + \frac{e}{c}A_{oy})^2]^{1/2} \tag{15}$$

The exact solution of Eq. 10 is

$$g^{(1)}(\alpha,\beta,u,z) = \exp[i\int_0^z \frac{\omega}{v_z(\alpha,\beta,u,\bar{z})} d\bar{z}] \int_0^z \exp[-i\int_0^{z''} \frac{\omega}{v_z(\alpha,\beta,u,\bar{z}'')} d\bar{z}''] \frac{H(\alpha,\beta,u,z'')}{v_z(\alpha,\beta,u,z'')} dz'' \tag{16}$$

where $H(\alpha,\beta,u,z)$ stands for the right hand side of Eq. 10, and we assumed $g^{(1)}(\alpha,\beta,u,o) = 0$. (no prebunching of the electron beam). Substitution of (16) in (12) gives

$$\tilde{J}_{\pm}(z) = -i\frac{e^2}{2c}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{z} d\alpha\, d\beta\, du\, dz'' \frac{\exp[-i\int_0^z \frac{\omega}{v_z(\alpha,\beta,u,\bar{z})} d\bar{z}]}{p_z(\alpha,\beta,u,z)} \frac{\exp[-i\int_0^{z''} \frac{\omega}{v_z(\alpha,\beta,u,\bar{z}'')} d\bar{z}'']}{p_z(\alpha,\beta,u,z'')}$$

$$p_{\pm}(\alpha,\beta,u,z)\,[p_-(\alpha,\beta,u,z'')\tilde{A}_+(z'') + p_+(\alpha,\beta,u,z'')\tilde{A}_-(z'')]\,\frac{\partial}{\partial u}\, g^{(o)}(\alpha,\beta,u) \quad . \tag{17}$$

The electromagnetic field induced by the current is given by

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2}\right) \tilde{A}_{\pm}\,(z) = -\,\frac{4\pi}{c}\,\tilde{J}_{\pm}\,(z) \tag{18}$$

This second order equation can be reduced to a first order equation if we assume that there is only a forward going wave propagating with a wave number $\sim\omega/c$, and there is no coupling to a backward going electromagnetic wave. We can then reduce (18) into

$$\left(\frac{\partial}{\partial z} - i\,\frac{\omega}{c}\right)\tilde{A}_{\pm}\,(z) = \frac{2\pi i}{\omega}\,\tilde{J}_{\pm}\,(z) \tag{19}$$

which can be readily transformed into an integral equation:

$$\tilde{A}_{\pm}\,(z) = \tilde{A}_{\pm}\,(o)e^{i\frac{\omega}{c}z} + \frac{2\pi i}{\omega}\,e^{i\frac{\omega}{c}z}\int_{o}^{z} e^{-i\frac{\omega}{c}z'}\,\tilde{J}_{\pm}\,(z')dz' \tag{20}$$

Equations (17) and (20) are a set of two coupled integral equations, which can be used to solve for the linear evolution of the electromagnetic field. These equations can be solved by an iterative process. To zero order (no interaction) we have $\tilde{A}_{\pm}\,(z) = \tilde{A}_{\pm}\,(o)\exp(i\omega z/c)$. We substitute the zero order iteration in (17) which is in turn substituted in (20) to yield the expression for the electromagnetic field to first order in $\tilde{A}_{\pm}\,(o)$. The iterative process may be continued in this way to higher orders.

At present we assume small gain and find $\tilde{A}_{\pm}(z)$ to first order:

$$\tilde{A}_{\pm}(z) = \{ \tilde{A}_{\pm}(o) + \frac{\pi e^2}{c} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{z} d\alpha d\beta du dz' \int_{0}^{z'} dz'' \frac{\partial}{\partial u} g^{(o)}(\alpha,\beta,u)$$

$$\frac{\exp[i\int_{0}^{z'}(\frac{\omega}{v_z'} - \frac{\omega}{c})d\bar{z}']}{p_z'} \; \frac{\exp[-i\int_{0}^{z''}(\frac{\omega}{v_z''} - \frac{\omega}{c})d\bar{z}'']}{p_z''} \; p_{\pm}'[p_{-}'' \tilde{A}_{+}(o) + p_{+}'' \tilde{A}_{-}(o)]\} e^{i\frac{\omega}{c} z} \tag{21}$$

The primed and double primed parameters p_z, $p_{\pm}$ are all dependent on α,β, u and z' or z'' respectively, $v_z' = v_z(\alpha,\beta,u,\bar{z}')$, $v_z'' = v_z(\alpha,\beta,u,\bar{z}'')$. The Poynting vector power density of the fields in Eqs. 8, 9, is found to be given by:

$$\underline{S} = \hat{e}_z \frac{i}{8} \frac{\omega}{c} [\tilde{A}_{+}(z) \frac{d}{dz} A_{+}^{*}(z) + \tilde{A}_{-}(z) \frac{d}{dz} A_{-}^{*}(z)] + \text{c.c.} \tag{22}$$

We assume that in (21) the expression in the curled parenthesis varies very slowly with z in comparison to $\exp(i\omega z/c)$, hence instead of (22) we can write

$$\underline{S} = \hat{e}_z \frac{1}{4} (\frac{\omega}{c})^2 [|\tilde{A}_{+}(z)|^2 + |\tilde{A}_{-}(z)|^2] . \tag{23}$$

Let us assume now that the incident electromagnetic wave is right hand circularly polarized ($\tilde{A}_{-}(o) = 0$). The power gain of this wave $\tilde{A}_{+}(z)$ after an interaction distance ℓ is

$$\frac{\Delta P_+(\ell)}{P_+(o)} = \left|\frac{A_+(\ell)}{A_+(o)}\right|^2 - 1 \simeq \frac{\pi e^2}{c} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\ell} d\alpha d\beta du dz' \int_{0}^{z'} dz''$$

$$\frac{\partial}{\partial u} g^{(o)}(\alpha,\beta,u) \frac{p_+'}{p_z'} \exp\left[i\int_0^{z'} \left(\frac{\omega}{v_z'} - \frac{\omega}{c}\right) d\bar{z}'\right] \frac{p_-''}{p_z''} \exp\left[-i\int_0^{z''} \left(\frac{\omega}{v_z''} - \frac{\omega}{c}\right) d\bar{z}''\right] + c.c. \tag{24}$$

This is a general expression for the linear gain in the low gain limit which within the limits of the assumptions used during the derivation is applicable for arbitrary three dimensional beam velocity distribution and arbitrary transverse field pump which can be written in the form (3).

We may now apply Eq. 24 to the case of the Stanford free electron laser experiment [5] where we assume a pure right hand helical magnetic field

$$A_o(z) = A_o \cos k_o z \, \hat{e}_x + A_o \sin k_o z \, \hat{e}_y \tag{25}$$

We also assume that the electron beam has negligible transverse velocity spread, and zero transverse canonical momentum. The equilibrium electron distribution function can, therefore, be written as

$$g^{(o)}(\alpha,\beta,u) = n_o \delta(\alpha)\delta(\beta) g_o(u) \tag{26}$$

In this case we get from (14), (15)

$$p_{\pm} = -\frac{e}{c} A_o e^{\mp i k_o z} \tag{27}$$

$$p_z = [u^2 - (\frac{e}{c} A_o)^2]^{1/2} \tag{28}$$

and we see that p_z , v_z are no longer functions of z. Equation (24) simplifies into

$$\frac{\Delta P_+(\ell)}{P_+(o)} = \frac{\omega_p^2}{2} \frac{m}{c} \left(\frac{eA_o}{c}\right)^2 \int_o^\infty \int_o^\ell \frac{du}{p_z^2} dz' \int_o^{z'} d\tau \frac{dg_o(u)}{du} \cos\left(\frac{\omega}{v_z} - \frac{\omega}{c} - k_o\right)\tau =$$

$$= \frac{\omega_p^2}{4} \frac{m}{c} \left(\frac{eA_o}{c}\right)^2 \int_o^\infty \frac{du}{p_z^2} \frac{dg_o(u)}{du} \frac{\sin^2 \frac{1}{2}\left(\frac{\omega}{v_z} - \frac{\omega}{c} - k_o\right)\ell}{\left[\frac{1}{2}\left(\frac{\omega}{v_z} - \frac{\omega}{c} - k_o\right)\right]^2} \tag{29}$$

In the cold beam limit

$$\frac{v_{zth}}{v_{zo}} << \beta_{zo} \frac{\lambda}{\ell} \tag{30}$$

where v_{zth} is the beam velocity spread in the z direction and $\beta_{zo} = v_{zo}/c$, we may substitute into (29) $g_o(u) = \delta(u-u_o)$. Equation 29 then reduces to the conventional gain expression [1-4].

$$\frac{\Delta P_+(\ell)}{P_+(o)} = \frac{1}{8} \frac{1}{\gamma_o \gamma_{zo}^2} \left(\frac{\omega_p}{v_{zo}}\right)^2 \left(\frac{p_{\perp o}}{p_{zo}}\right)^2 \frac{u_o}{p_{zo}} \frac{\omega}{c}\ell^3 \frac{d}{d\theta_o} \left(\frac{\sin \theta_o}{\theta_o}\right)^2 \tag{31}$$

where

$$\theta_o \equiv \left(\frac{\omega}{v_{zo}} - \frac{\omega}{c} - k_o\right) \ell/2 \tag{32}$$

$$p_{\perp o} \equiv \frac{e}{c} A_o = \frac{e}{c} B_o/k_o \tag{33}$$

$$p_{zo} \equiv \left(u_o^2 - p_{\perp o}^2\right)^{1/2} \tag{34}$$

$$\gamma_{zo} \equiv (1 - v_{zo}^2/c^2)^{-1/2} \tag{35}$$

The gain curve (31) obtains its maximum value at $\theta_o = -1.3$.

It is worth noting that the simplified form of Eq. (29) is due to the fact that in the limit of an ideal helical pump field and $\alpha = \beta = o$, the longitudinal momentum p_z (Eq. 28) is a constant of the motion and is independent on z. If for instance the pump field was not helical but linearly polarized ($A_o(z) = A_o \cos k_o z \hat{e}_x$ instead of (25)), then p_z and v_z in Eq. 24 would be still dependent on z in a periodic way (see 15). The integration over z' z'' is then not immediate. We will not elaborate on this case beyond noting that the gain spectrum for this pump will be rich with harmonics.

We now will examine the effect on the gain of transverse momentum spread of the electron beam and of irregularities in the helical magnetic pump field. Of course, if the magnetic pump field $\underline{B}_o(z)$ is known exactly (e.g., measured directly in a specific experiment), then it is possible to find $A_o(z)$ by integrating Eq. 2. If also the beam velocity distribution $g_o(\alpha,\beta,u)$ is known explicitly, then the gain can be calculated "exactly" using the general gain expression (24). In practice we are interested in estimating the effects on the gain due to a transverse momentum spread and irregularities in the helical pump field even if we do not know exactly the beam velocity distribution and the exact form for the pump field. We will, therefore, find the conditions in which the effects on the gain of transverse momentum spread and of irregularities in the pump field are significant enough so that (26) and (25) and consequently (29), (31) are not valid.

Irregularities in the helical field can be described as amplitude and phase modulation of the ideal helical field, so that

$$\underline{A}_o(z) = A_o(z)\cos[k_o z + \phi(z)]\hat{e}_x + A_o(z)\sin[k_o z + \phi(z)]\hat{e}_y \tag{36}$$

instead of (25). The amplitude $A_o(z)$ is randomly modulated around an average value $\langle A_o(z)\rangle = A_o$. The term $\phi(z)$ is a random phase deviation of the vector potential field $\underline{A}_o(z)$. It should be pointed out here that $\phi(z)$ and $A_o(z)$ are not related in a simple way to the phase and amplitude modulation of the magnetic field $\underline{B}_o(z)$. They can be evaluated only after integrating Eq. 2.

With Eq. (36) and without assuming (26) we have from (14)

$$p_\pm = \alpha \mp i\beta - \frac{e}{c} A_o(z) e^{\mp i[k_o z + \phi(z)]} \tag{37}$$

and by expanding (15) to first order in $\delta A_o(z)$, α and β

$$\frac{1}{p_z(z)} \simeq \frac{1}{p_z(u)} \; \{ 1 + \frac{p_{\perp o}}{p_z^2}$$

$$[\delta p_{\perp o}(z) + \alpha\cos(k_o z + \phi(z)) + \beta\sin(k_o z + \phi(z))]\} \tag{38}$$

$$\frac{1}{v_z(z)} \simeq \frac{1}{v_z(u)} \; \{1 + \frac{p_{\perp o}}{p_z^2}$$

$$[\delta p_{\perp o}(z) + \alpha\cos(k_o z + \phi(z)) + \beta\sin(k_o z + \phi(z))]\} \tag{39}$$

where

$$A_o(z) \equiv A_o + \delta A_o(z) \tag{40}$$

$$\delta p_{\perp o}(z) \equiv \frac{e}{c}\,\delta A_o(z) \tag{41}$$

We see immediately that we may set $\alpha = \beta = o$ in Eq. 37 and set

$$p_{\pm} \simeq \frac{e}{c} A_o(z) e^{\mp i[k_o z + \phi(z)]} \tag{42}$$

This is true for two reasons: (a) The transverse momentum spread α_{th} is assumed to satisfy $\alpha_{th} << p_{\perp o}$ and, therefore, the additive contribution of $\alpha \mp i\beta$ to (37) and in turn to (24) is negligible, (b) α and β do not have periodic z dependence like $A_{\pm}(z)$, therefore, they will not contribute a synchronous term in the integrand of (24) and thus will have a negligible contribution. On the other hand we cannot set $\alpha = \beta = o$ in (39) without careful examination, because $v_z(z)$ appears in the exponents of Eq. 24 and small deviations in $v_z(z)$ may have a significant effect on the integral and spoil the synchronism condition which led to the "resonant" expression in Eq. 29 or 31. As for $p_z(z)$ (Eq. 38) we can (to zero order) approximate it by $p_z(z) = p_z(u)$ in (24). Again the reason is that the contribution of the first order correction is additive and small.

We then get from (24)

$$\frac{\Delta P_\perp(\ell)}{P_\perp(o)} = 2\frac{\pi \ell^2}{c} \left(\frac{e}{c} A_o\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{o}^{\infty}\int_{o}^{\ell} d\alpha d\beta du dz' \int_{o}^{z'} dz'' \frac{1}{p_z^2(u)} \frac{\partial}{\partial u} g^{(o)}(\alpha,\beta,u)$$

$$\cos\left[\left(\frac{\omega}{v_z(u)} - \frac{\omega}{c} - k_o\right)(z' - z'') + \psi(z') - \psi(z'')\right] \tag{43}$$

where

$$\psi(z) = \frac{\omega}{v_z(u)} \frac{p_{\perp o}}{p_z^2} \int_{o}^{z'} \{\delta p_{\perp o}(\bar{z}) + \alpha\cos[k_o\bar{z} + \phi(\bar{z})] + \beta\sin[k_o\bar{z} + \phi(\bar{z})]\}d\bar{z} - \phi(z) \tag{44}$$

Using the last expressions, we will first examine the effect of transverse velocity spread on the gain. For the sake of simplicity we assume now an ideal helical field $A_o(z) = A_o$, $\phi(z) = 0$ and a Maxwellian transverse momentum spread of the electron beam.

$$g^{(o)}(\alpha,\beta,u) = \frac{n_o}{\pi\alpha_{th}^2} e^{-(\alpha^2 + \beta^2)/\alpha^2_{th}} g_o(u) \tag{45}$$

$$\frac{\Delta P_+(\ell)}{P_+(o)} = \frac{\omega_p^2}{2} \frac{m}{c} \left(\frac{eA_o}{c}\right)^2 \int_o^\infty\int_o^\ell \frac{du}{p_z^2} dz' \int_o^{z'} d\tau e^{-\left[\frac{\omega}{v_z k_o} \frac{p_{\perp o}}{p_z} \frac{\alpha_{th}}{p_z} \sin\frac{1}{2} k_o\tau\right]^2} \cos\left(\frac{\omega}{v_z} - \frac{\omega}{c} - k_o\right)\tau \tag{46}$$

Further investigation of the integral in (46) will not presently be carried out. We only note that besides reducing the gain, the transverse momentum also generates harmonic frequencies in the gain spectrum. We may also find from (46) that the condition at which the neglect of the transverse momentum spread is allowed is $\frac{\omega}{v_{zo}k_o}\frac{p_{\perp o}}{p_{zo}}\frac{\alpha_{th}}{p_{zo}} << 1$. Using the synchronism condition $\frac{\omega}{c} \simeq \frac{\omega}{v_{zo}} - k_c$, this can be written as

$$\frac{\alpha_{th}}{p_{zo}} << \frac{1}{(1+\beta_{zo})\gamma_{zo}^2\beta_{\perp o}} \tag{47}$$

where $\beta_{\perp o} \equiv p_{\perp o}/p_{zo} = eB_o/(\gamma_o mcv_{zo}k_o)$. In the relativistic limit $(\gamma_o >> 1)$ this condition is

$$\frac{\mathcal{E}_{th}}{\mathcal{E}_o} << \frac{1}{2\gamma_{zo}^2\beta_{\perp o}} \tag{48}$$

When (47) or (48) are well satisfied we may set in (46) $\alpha_{th} = o$ and recover (29).

It is interesting to compare conditions (47,48) with condition (30) for the longitudinal momentum spread, which can be written in the forms:

$$\frac{p_{zth}}{p_{zo}} << \beta_{zo}\gamma_{zo}^2\frac{\lambda}{\ell} \tag{49}$$

or in the relativistic limit

$$\frac{\mathcal{E}_{zth}}{\mathcal{E}_o} << \gamma_{zo}^2\frac{\lambda}{\ell} \tag{50}$$

In the regime $\frac{\ell}{\lambda} \frac{1}{(1 + \beta_{zo})\beta_{zo}\beta_{\perp o}\gamma_{oz}^4} < 1$ $(\frac{\ell}{\lambda} \frac{1}{2\beta_{\perp o}\gamma_{oz}^4} < 1$ - relativistic) we find that the condition on the transverse momentum spread (47) is more stringent than the condition on the longitudinal spread. With high energy beams this may easily be the case and conditions (47,48) may be of practical concern. It is easy to verify, that condition (48) and (50) are reasonably well satisfied for the parameters of the Stanford experiment.

We now consider the effect of irregularities in the helical pump field, and this time we assume for simplicity $\alpha_{th} = 0$ (Eq. 26). Equation 43 then simplifies into

$$\frac{\Delta P_+(\ell)}{P_+(o)} = \omega_p^2 \frac{m}{c} \int_o^\infty \int_o^\ell du dz' \int_o^{z'} dz'' \frac{dg_o(u)}{du} \frac{p_{\perp o}(z')}{p_z(z')} \frac{p_{\perp o}(z'')}{p_z(z'')}$$

$$\cos\left[\left(\frac{\omega}{v_z} - \frac{\omega}{c} - k_o\right)(z' - z'') + \psi(z') - \psi(z'')\right] \quad (51)$$

where

$$\psi(z) = \frac{\omega}{v_z} \left(\frac{e}{c}\right)^2 \frac{A_o}{p_z^2} \int_o^z \delta A_o(\bar{z}) d\bar{z} - \phi(z) \quad (52)$$

Notice that the phase deviation $\psi(z)$ is effected not only by the phase deviation of the helical field vector potential $\phi(z)$ but also by the amplitude deviation $\delta A_o(z)$.

The condition that Eq. (51) be reduced to (29) and the effect of imperfections in the helical field be neglected is

$$|\psi(z)| << \pi \quad (53)$$

for any $o < z < \ell$. In this condition it is possible to expand the integrand in (49) to first order in $\psi(z)$ and the first order term will be small compared to the zero order term (29). Condition (53) may be quite a stringent condition when the interaction length is long. For example, to appreciate the effect of changes in the field amplitude $\delta A_o(z)$, assume that it is changing linearly throughout the interaction length ℓ, then inequality (53) gives

$$\left| \frac{\delta A_o(\ell)}{A_o} \right| << \frac{\beta_{zo}}{\beta_{\perp o}^2} \frac{\lambda}{\ell} \tag{54}$$

For $\lambda = 10\mu m$, $\ell = 4m$, $\beta_{\perp o} = 0.05$, $\beta_{zo} = 1$ this requires that

$$\left| \frac{\delta A_o(\ell)}{A_o} \right| << 10^{-3} .$$

We can now appreciate the significance of inaccuracies in the periodicity of the helical field. Small local deviations in the field period will not cause major change in the gain expression (29) as long as there is no cumulative phase deviation throughout the interaction length ℓ which violates (53). In other words, the long range coherence of the periodic structure is important. We may add that similar conditions probably would apply also to the problem of Compton and Raman scattering from electron beams [6,7] where the coherent length of the scattering wave may limit the obtainable gain. This may be an important consideration especially when the scattering (pump) wave is produced by a low coherence source like a high power laser or a wide band microwave source.

In the case where condition (53) is not satisfied the free electron laser gain curve may differ substantially from (31). For any specific laser structure with pump field which deviates from the ideal helical dependence (25), the integration of (51) would result different gain curve. A statistical analysis may be carried out to estimate the quantitative effect of random perturbations of the pump field on the laser gain [8]. Such an analysis can yield expressions for the average laser gain which for a static pump is essentially an ensemble average over many randomly perturbed free electron lasers. For an electromagnetic pump (stimulated Compton scattering) the statistical analysis would yield a time average of the laser gain.

Detailed statistical analysis of the laser gain in the regime where (53) does not hold is beyond the scope of the present article. We will only point out that the gain in this case depends basically on two parameters of the pump field random perturbation - the phase modulation index $<\psi^2(z)>$ and the pump coherence length ℓ_c. Whenever $<\psi^2(z)> << 1$ or $\ell/\ell_c << 1$ the average gain curve is close to (31). In the limit of short coherence length the average gain curve changes substantially and is reduced by a factor of about $(\ell/\ell_c)^2$ relative to the unperturbed free electron laser gain.

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